EE-580 Space Mechanisms

Exercise 1.1 - Energy of meteoroids

v.02

Problem Statement

The meteoroids on terrestrial orbit have an average density of 2.5 g/cm^3 and a speed of the order of 13km/s.
For this speed, calculate the diameter of a meteoroid (sphere), which energy, if transferred without loss to 200g of water at 20°C, would bring this water to:

1. 100°C
2. Totally vaporize it

Solution

Specific mass of the meteoroid [kg/m ^{3}]:

In[•]:=
```
ρ = 2500;
```

Mass of the meteoroid (sphere) [kg]:

In[•]:=
```
m[d_] := 4 π/3 (d/2)^3 ρ;
```

Speed of the meteoroid [m/s]:

In[•]:=
```
v = 13 × 10^3;
```

Kinetic energy of the meteoroid [J]:

In[•]:=
```
E_kin[d_] := m[d] v^2/2;
```

In[•]:=
```
Plot[E_kin[d], {d, 0, 20 × 10^-3}]
```
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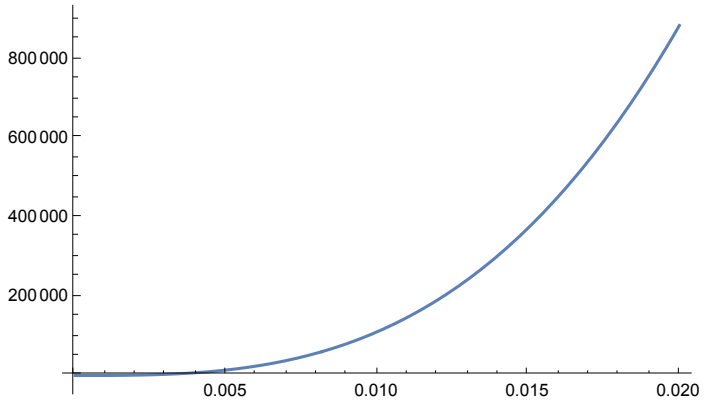
Out[•]=



Mass of water [kg]:

In[•]:=
```
m_water = 0.2;
```

Initial temperature of the water [°C]:

In[•]:=
```
T_ini = 20;
```

Water boiling temperature [°C]:

In[•]:=
```
T_boil = 100;
```

Specific heat of liquid water [J/(kg·K)]:

In[•]:=
```
c_liq = 4186;
```

Latent heat of vaporization of water [J/kg]:

In[•]:=
```
L_water = 2265 × 10^3;
```

Specific heat of water vapour [J/(kg·K)]:

In[•]:=
```
c_vap = 2010;
```

Heating of the water [°C]:
Energy conservation:

- *Under the water boiling temperature:* $E_{kin} = (T[d]\text{-}T_{ini}) \cdot c_{liq} \cdot m_{water}$
- *For vapour:* $E_{kin} = (T_{boil} - T_{ini}) \cdot c_{liq} \cdot m_{water} + (T[d]\text{-}T_{boil}) \cdot c_{vap} \cdot m_{water} + L_{water} \cdot m_{water}$
- *For boiling water: temperature is constant*

In[•]:=
```
T[d_] := { E_kin[d]/(c_liq m_water) + T_ini                                                  ,  E_kin[d]/(c_liq m_water) + T_ini < T_boil
         { (E_kin[d] - (T_boil - T_ini) c_liq m_water - L_water m_water)/(c_vap m_water) + T_boil ,  (E_kin[d] - (T_boil - T_ini) c_liq m_water - L_water m_water)/(c_vap m_water) > 0
         { T_boil                                                                            ,  True
```

Plot of the heating of the water T [°C] vs meteoroid diameter d [m]:

In[•]:=
```
Plot[T[d], {d, 0, 20 × 10^-3}, AxesLabel → {HoldForm[d in m], HoldForm[T in °C]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```
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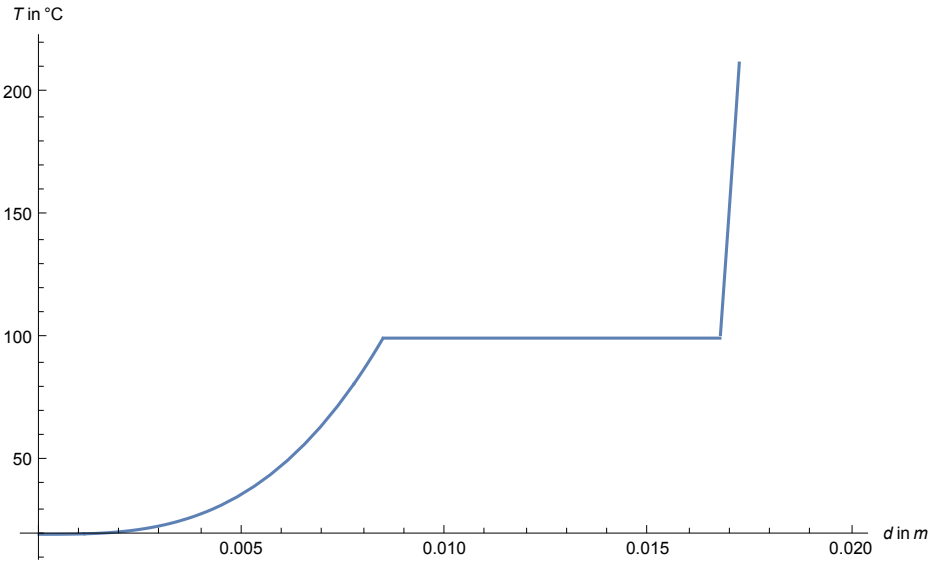
Out[•]=



Diameter of the meteoroid [m]:

1. Heating to 100°C:

In[•]:=
```
Reduce[E_kin[d] / (c_liq m_water) + T_ini == T_boil, d, Reals]
```
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$$d == 0.00846008$$

2. Totally vaporize the water

In[•]:=
```
Reduce[(E_kin[d] - (T_boil - T_ini) c_liq m_water - L_water m_water) / (c_vap m_water) == 0, d, Reals]
```
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Out[•]=

$$d == 0.0167518$$